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A NEW APPROACH TO BLIND CHANNEL  
IDENTIFICATION FOR OFDM SYSTEMS

FINAL TECHNICAL REPORT  
BY

DR MOUNIR GHOGHO  
SEPTEMBER 2003

UNITED STATES ARMY  
EUROPEAN RESEARCH OFFICE OF THE U.S. ARMY  
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## Abstract

*Orthogonal frequency division multiplexing (OFDM) is, with substantial progress in digital signal processing, becoming an important part of the telecommunications arena. The most appealing feature of OFDM is the simplicity of the receiver design due to the efficiency with which OFDM can cope with the effects of frequency-selective multipath channels. Here, we address the problem of channel estimation for OFDM systems. Exploiting receive antenna diversity, a second-order statistics-based (SOS) blind technique is proposed. Our method differs from the existing SOS-based techniques in that channel estimation is carried out using the frequency domain (i.e., post-FFT) signals whereas existing methods use the time domain signals (i.e., pre-FFT). In the proposed method, channel identifiability is guaranteed regardless of the channels zeros locations, so long as any roots common to all the diversity channels are on the unit circle. Only short data records are required to achieve good performance. For PSK transmission, this method enables channel estimation even from a single OFDM symbol at high SNR. Further, when only a small number of subcarriers can be used for channel estimation because of computational complexity, we determine the optimal set of subcarriers in terms of estimation accuracy.*

**keywords:** OFDM, Multipath channel, Antenna diversity, Identification, Estimation, Phase-Shift-Keying, Quadrature-Amplitude-Modulation.

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# Chapter 1

## OFDM Systems

Orthogonal frequency division multiplexing (OFDM) has become the standard of choice for wireless LAN's such as HIPERLAN/2 and IEEE 802.11a; it has been adopted in Europe for Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB), MMAC in Japan, and fixed wireless; and is being considered for several IEEE 802.11 and 802.16 standards, including wideband Metropolitan Area Networks (MAN) [1]. The popularity of OFDM stems from its ability to transform a wideband frequency selective channel to a set of parallel flat-fading narrowband channels, which substantially simplifies the channel equalization problem. Because of the time-frequency granularity that it offers, OFDM appears to be a natural solution when the available spectrum is not contiguous, for overlay systems, and to cope with issues such as narrowband jamming. In the multi-user context, this granularity also accommodates variable quality-of-service (QoS) requirements and bursty data.

In this report, we review the basics of OFDM, and study the channel estimation problem. Clearly, issues such as timing and carrier recovery are important, but will not be treated in this chapter. Channel estimation usually consists of an acquisition phase followed by a tracking phase. In the acquisition phase, initial (often coarse) estimates of the channel parameters are obtained. Finer estimates, as well as tracking of small time-variations, of these parameters are acquired during the tracking phase.

The origins of OFDM can be traced back to a 1957 multi-carrier HF analog modem (the Kineplex), but it was only with digital implementations via the FFT and the introduction of the cyclic prefix, both by Weinstein and Ebert [2], and seminal analyzes by Cimini [3], that OFDM became practical. Several recent books and papers on OFDM cover different aspects of signal and system design [4, 5, 6, 7, 8].

*Notation:* We will let  $x(n, m)$  denote the  $m$ -th element of the vector  $\mathbf{x}(n)$ . Superscripts  $\mathcal{H}$  and  $T$  will denote conjugate transposition and transposition.  $\mathbf{F}$  will denote the FFT matrix with  $(k, m)$ th entry  $\frac{1}{\sqrt{M}} \exp(-j2\pi km/M)$ .  $Tr$  denotes the trace operator.

## 1.1 Basics of OFDM

Assume that the information bearing symbols are to be transmitted at the rate of  $R$  symbols per second over a multipath propagation channel. The duration of each symbol is therefore  $T_s = 1/R$ . If the delay spread<sup>1</sup>,  $\tau_{max}$ , of the channel is larger than about 10% of the symbol duration, then the received signal may suffer from significant inter-symbol interference (ISI), which can drastically increase the symbol-error-rate (SER)<sup>2</sup> unless counter-measures are undertaken. Such a channel is said to be dispersive or frequency-selective. There are two main approaches to cope with such channels. The first approach is to use a single-carrier system with an equalizer at the receiver to compensate for the ISI, which spans  $\lceil \tau_{max}/T_s \rceil$  symbols. The implementation of the equalizer may become very challenging for channels with large delay spreads, and at higher data rates. The second approach is based on multicarrier modulation, such as orthogonal frequency division multiplexing (OFDM). Here, we focus on the latter approach.

The operational principle of an OFDM system is that the available bandwidth is divided into a large number of sub-bands, over each of which the wireless channel can be considered non-dispersive or flat-fading. The original data stream at rate  $R$  is split into  $M$  parallel data streams, each at rate  $R/M$ . The symbol duration,  $T$ , for these parallel data streams is therefore increased by a factor of  $M$ , i.e.,  $T = MT_s$ . Conceptually, each of the data streams modulates a carrier with a different frequency and the resulting signals are transmitted simultaneously (in reality, a single modulator is used, as we discuss later). Correspondingly, the receiver consists of  $M$  parallel receiver paths. Due to the increased symbol duration, the ISI over each channel is reduced to  $\lceil \tau_{max}/(MT_s) \rceil$  symbols. Thus, an advantage of OFDM is that, for frequency-selective fading channels, the OFDM symbols are less affected by channel fades than are single-carrier transmitted symbols. This is due to the increased symbol duration in an OFDM system. While many symbols during a channel fade might be lost in a single-carrier system, the symbols of an OFDM system can still be correctly detected as only a fraction of each symbol might be affected by the fade. On the other hand, if the channel is time-selective, i.e., the channel impulse response varies significantly within the OFDM symbol period, then the channel matrix is no longer Toeplitz, and conventional OFDM would fail.

Since multicarrier modulation is based on a block transmission scheme, measures have to be taken to avoid or compensate for interblock interference (IBI), which contributes to the overall ISI. OFDM systems can be categorized in the way they handle IBI. In the most popular system, a guard time is introduced between consecutive OFDM symbols as a *cyclic prefix* (CP), i.e., the tail end of the OFDM symbol is prefixed. The length of the cyclic prefix is chosen to be larger than the expected delay spread; after proper time synchronization, the receiver discards the CP and thus the IBI is eliminated. Time-

---

<sup>1</sup>The delay spread is a measure of the delay of the longest path (or last echo) with respect to that of the earliest path (or first arrival) typically the root-mean-square (RMS) delay spread is used.

<sup>2</sup>The symbol error rate is the rate of errors made during the symbol detection process at the receiver.

guarding by zero-padding the OFDM symbols has also been proposed in [9, 10]. The issue here is one of turning the transmitter on and off and increased receiver complexity vs. the increased SNR and decreased SER. Comparisons between cyclic-prefixing and zero-padding OFDM systems may be found in [11].

To achieve high resilience against channel dispersion, a large number of subcarriers is required. However, the implementation of a large number of modulators and demodulators can be very complex, both in terms of the physical size of the radio, as well as the difficulty of locking in multiple oscillators. This complexity can be significantly reduced by digitally performing the modulation and demodulation using the discrete Fourier transform (DFT) and its inverse (IDFT) [2]. An efficient implementation of the DFT may be obtained by any of the available fast Fourier transform (FFT) algorithms.

The choice of the OFDM parameters is a tradeoff between various, often conflicting, requirements. The length of the CP is dictated by the delay spread of the channel. Introduction of the CP entails a reduction in rate (or wasted bandwidth), as well as a SNR loss; to minimize these inefficiencies, the number of subcarriers,  $M$ , should be large. However, a large number of subcarriers induces a high implementation complexity, increased sensitivity to frequency offset and phase noise (since the subcarriers get closer to each other as  $M$  increases) and an increased peak-to-average power ratio (PAPR).  $M$  is dictated by concerns regarding practical FFT sizes as well as the coherence time of the channel. We will not address the issue of practical choice of OFDM parameters here; we refer the reader to [3, 6, 8]. In this chapter, we address the crucial issue of CFO estimation.

We confine our attention to OFDM; generalized schemes which also convert frequency-selective channels into a bank of flat-fading channels exist [12]; see also [13]. In [12], the OFDM scheme is shown to be the optimal precoder which uses a cyclic prefix. In a multiple-user setting, OFDMA has been shown to be the optimal scheme in the sense of maximizing the SNR for each user [14].

### 1.1.1 OFDM Modulation

OFDM modulation consists of  $M$  (usually a power of 2) sub-carriers, equi-spaced at a separation of  $\Delta f = B/M$ , where  $B$  is the total system bandwidth. All sub-carriers are mutually orthogonal over a time interval of length  $T = 1/\Delta f$ . Each sub-carrier is modulated independently with information-bearing symbols (this does not preclude coding across the sub-carriers). Each OFDM block is preceded by a CP whose duration is usually longer than the delay spread of the propagation channel, so that IBI can be eliminated at the receiver, without affecting the orthogonality of the sub-carriers. Practical OFDM systems are not fully loaded in order to avoid interference between adjacent OFDM systems: some of the sub-carriers at the edges of the OFDM block are not modulated; these subcarriers are referred to as virtual subcarriers (VSC). The number of these VSC is dictated by system design requirements and is, in general, about 10% of  $M$ . Some subcarriers,

other than the VSC, may also be deactivated. For example, when channel state information (CSI) is available to the transmitter, subcarriers experiencing deep fades will be left unmodulated. Further, synchronization preambles are often made by nulling a large number of subcarriers. Indeed, a preamble consisting of a repetition of two identical slots is obtained by nulling all the odd subcarriers [?]. Here, deactivated subcarriers will be referred to as null-subcarriers (NSC). The set of NSC includes the VSC, whose placement and number are imposed by system design; the number and placement of the remaining NSC are controlled by the system user, and could vary across the OFDM symbols. Let  $\mathcal{M} = \{0, \dots, M-1\}$  denote the entire set of sub-carriers, and let  $\mathcal{K}_n$  (resp.  $\mathcal{Z}_n$ ) denote the subset of  $\mathcal{M}$  that contains the  $K_n$  (resp.  $Z_n$ ) modulated (resp. null) subcarriers during the  $n$ th OFDM symbol or block.

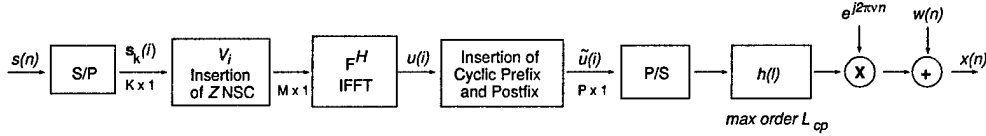


Figure 1.1: Discrete-time complex baseband representation

The discrete-time block diagram of a standard OFDM system is depicted in Fig. 1.1. The vector modulating the entire set of subcarriers during the  $n$ th block can then be expressed as  $\mathbf{s}(n) := \mathbf{V}_n \mathbf{s}_{\mathcal{K}}(n)$ , where  $\mathbf{s}_{\mathcal{K}}(n)$  is the  $K_n$ -element vector of symbols transmitted on the activated subcarriers, and  $\mathbf{V}_n$  is the  $M \times K_n$  matrix whose  $(m, \ell)$ -th entry is one if the  $\ell$ th symbol is transmitted on the  $m$ -th subcarrier during the  $n$ th OFDM block, and is zero otherwise. Matrix  $\mathbf{V}_n$  is a full-rank submatrix of an  $(M \times M)$  permutation matrix. We assume without loss of generality that the symbols are zero mean and have unit variance, i.e.,  $E|s(n, m)|^2 = 1$ . The  $(M \times 1)$  data block  $\mathbf{s}(n)$  is first precoded by the IFFT matrix  $\mathbf{F}^H$ . The resulting  $(M \times 1)$  vector  $\mathbf{u}(n) = \beta_n \mathbf{F}^H \mathbf{s}(n)$  is called the time-domain block vector, or the time domain OFDM symbol. We have also introduced a normalization parameter  $\beta_n := \sqrt{M/K_n}$  to ensure that the transmitted power is kept constant regardless of  $K_n$ , the number of active subcarriers. Next, a CP of length  $L_{cp}$  is inserted by replicating the last  $L_{cp}$  elements of each block in the front. The redundant block vector can be expressed as

$$\tilde{\mathbf{u}}(n) = [u(n, M - L_{cp}), u(n, M - L_{cp} + 1), \dots, u(n, M - 1), u(n, 0), \dots, u(n, M - 1)]^T.$$

The  $P (= M + L_{cp})$  samples of each block are then pulse shaped, upconverted to the carrier frequency, and transmitted sequentially through the channel.



### 1.1.2 Demodulation

We model the frequency-selective channel as an FIR filter with channel impulse response (CIR)  $\mathbf{h} = [h_0, \dots, h_L]^T$  where  $L$  is the channel order. In practice, the system is *usually* designed such that  $L \leq L_{cp} \leq M$ . We assume that the CIR is time-invariant over  $N \geq 1$  consecutive symbol blocks, but could vary from one set of  $N$  blocks to the next.

The received signal is downconverted to baseband and sampled at the rate of  $P$  samples per extended OFDM symbol. We will index these samples by  $[-L_{cp}, \dots, M-1]$ . We will assume that time synchronization has been achieved. Discarding the samples  $\ell = -L_{cp}, \dots, -1$  is known as discarding the cyclic prefix. The noise-free received signal corresponding to the  $n$ th OFDM symbol,  $\tilde{\mathbf{u}}(n)$ , can be written as

$$x(n, k) = \sum_{\ell=0}^L h_{\ell} \tilde{u}(n, k - \ell),$$

for  $k = 0, \dots, M-1$ . Recall that with the insertion of CP, we have  $\tilde{u}(n, \ell) = u(n, M + \ell)$  for  $\ell = -L_{cp}, \dots, -1$ , and  $\tilde{u}(n, \ell) = u(n, \ell)$  for  $\ell = 0, \dots, M-1$ . Then, collecting samples,  $k = 0, \dots, M-1$ , of  $x(n, k)$ , we obtain

$$\begin{aligned} \begin{bmatrix} x(n, 0) \\ \vdots \\ x(n, M-1) \end{bmatrix} &= \begin{bmatrix} h_0 & 0 & \dots & 0 & 0 & \dots & 0 \\ h_1 & h_0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h_{L-1} & h_{L-2} & \dots & h_0 & 0 & \dots & 0 \\ h_L & h_{L-1} & \dots & h_1 & h_0 & \mathbf{0} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \end{bmatrix} \begin{bmatrix} u(n, 0) \\ u(n, 1) \\ \vdots \\ u(n, M-1) \end{bmatrix} \\ &+ \begin{bmatrix} h_1 & h_2 & \dots & h_{L-1} & h_L & 0 & \dots & 0 \\ h_2 & h_3 & \dots & h_L & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_L & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} u(n, M-1) \\ u(n, M-2) \\ \vdots \\ u(n, M-L_{cp}) \end{bmatrix} \\ &= \begin{bmatrix} h_0 & 0 & \dots & 0 & 0 & \mathbf{0} & h_L & h_{L-1} & \dots & h_1 \\ h_1 & h_0 & \dots & 0 & 0 & \mathbf{0} & 0 & h_L & \dots & h_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h_{L-1} & h_{L-2} & \dots & h_0 & 0 & \mathbf{0} & 0 & 0 & \dots & h_L \\ h_L & h_{L-1} & \dots & h_1 & h_0 & \mathbf{0} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \end{bmatrix} \begin{bmatrix} u(n, 0) \\ u(n, 1) \\ \vdots \\ u(n, M-1) \end{bmatrix} \\ &\mathbf{x}(n) = \mathbf{H}_c \mathbf{u}(n) \end{aligned} \quad (1.1)$$

Notice that the matrix  $\mathbf{H}_c$  is circulant with first column,  $[h_0, h_1, \dots, h_L, 0, \dots, 0]^T$ .

Under the narrowband assumption, the noise corrupted counterpart of (1.1) becomes (assuming no carrier offset i.e.  $\nu = 0$  in Fig. 1)

$$\mathbf{x}(n) = \beta_n \mathbf{H}_c \mathbf{F}^H \mathbf{s}(n) + \mathbf{n}(n) \quad (1.2)$$

where  $\mathbf{H}_c$  is the  $(M \times M)$  circulant matrix defined in (1.1), and  $\mathbf{n}(n)$  is the  $M \times 1$  noise vector, which is assumed to be zero mean circular Gaussian with covariance matrix  $\sigma^2 \mathbf{I}$ .

Demodulation is based on the well-known property that any circulant matrix can be diagonalized by pre-multiplication by the FFT matrix and post-multiplication by the IFFT matrix. Let  $H_k := \sum_{l=0}^L h_l e^{-j2\pi kl/M}$ , denote the frequency response of the channel at frequency  $2\pi k/M$ , and let  $\mathbf{H} := \text{diag}(H_0, \dots, H_{M-1})$ . Then, the signal in (2.1) can be rewritten as

$$\begin{aligned} \mathbf{x}(n) &= \beta_n \mathbf{F}^H \mathbf{H} \mathbf{F} \mathbf{s}(n) + \mathbf{n}(n) \\ &= \beta_n \mathbf{F}^H \mathbf{H} \mathbf{s}(n) + \mathbf{n}(n) . \end{aligned} \quad (1.3)$$

Therefore, after FFT processing, the so-called frequency-domain received symbol blocks are obtained as

$$\mathbf{y}(n) = \mathbf{F} \mathbf{x}(n) = \beta_n \mathbf{F} \mathbf{F}^H \mathbf{H} \mathbf{s}(n) + \boldsymbol{\eta}(n) \quad (1.4)$$

where  $\boldsymbol{\eta}(n) = \mathbf{F} \mathbf{n}(n)$  is again AWGN with covariance matrix  $\sigma^2 \mathbf{I}$ . In the absence of carrier offset, the frequency-domain blocks are then obtained as

$$\mathbf{y}(n) = \beta_n \mathbf{H} \mathbf{s}(n) + \boldsymbol{\eta}(n) . \quad (1.5)$$

The effect of the frequency-selective channel on the OFDM signal is completely captured by scalar multiplications of the data symbols by the frequency responses of the channel at the subcarrier frequencies. Further, demodulation at the receiver does not color the additive noise. If none of the channel zeros coincides with an activated subcarrier, maximum likelihood detection of the symbols is straightforward. Zero-forcing and MMSE equalizers can be applied on a per-carrier basis. From eq. (1.5), we see that under the constraint of constant transmitted power, the presence of NSC (i.e.,  $\beta_n = M/K_n > 1$ ) implies a higher local SNR<sup>3</sup> at the modulated subcarriers at the expense of bandwidth efficiency. But, for frequency-selective channels, the local SNR can vary significantly across the subcarriers. Information transmitted on a subcarrier that is experiencing a deep fade (i.e., low SNR) could be lost, i.e., frequency selectivity of the channel could degrade BER. There are two major techniques to mitigate this problem. The first approach is to code across the subcarriers, typically by using a convolutional code. This improves SER at the expense of reduced rate, but does not require CSI at the transmitter. The second approach is based on power or bit loading techniques, and assumes that CSI is available at the transmitter. System capacity can be maximized by adapting the powers or the bit loads of the different subcarriers to the channel. In the case of bit loading, the constellation sizes of the symbols transmitted on the different subcarriers could be adjusted according to the corresponding SNRs using a water-filling method [15]. This is the typical scenario in discrete multitone (DMT), as the wired version of OFDM is called [16].

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<sup>3</sup>The local SNR at subcarrier  $k$  is defined as  $(M/K_n)(|H_k|^2/\sigma^2)$

## 1.2 Channel Estimation

As mentioned previously, the main advantage of OFDM systems is the reduced complexity of the equalizer at the receiver, which allows for inexpensive hardware implementation. Since OFDM transforms a frequency-selective channel into parallel flat-fading subchannels, a bank of one-tap equalizers suffices. Clearly, the channel needs to be estimated in order to design the equalizer. Channel identification may also be used for shortening the channel impulse response and determining power loading at the transmitter (if channel state information is available at the transmitter, channel nulls can be avoided by not transmitting symbols on those subcarriers).

Traditionally, channel estimation is carried out using pilot symbols. Because they save bandwidth and are capable of tracking (slow) channel variations, blind channel estimation and equalization methods are well motivated. Numerous blind channel identification algorithms for OFDM with redundancy<sup>4</sup> have recently been developed. Some of these methods are based on the cyclostationarity of oversampled OFDM signals [28], while others are based on subspace decompositions that exploit either the redundancy introduced by the cyclic prefix (CP) [29], or require changes to the current systems such as the use of precoding [30], or zero-padding [31]. Some of these methods require many OFDM symbols for channel estimation, so that they have to be modified for use over rapidly fading channels. Moreover, in subspace methods for CP-OFDM, channel identifiability is not guaranteed if the channel has nulls on the subcarriers [29]. A frequency domain method based on the finite-alphabet of the information-bearing symbols was developed in [32]. Blind channel estimation of single-input-multiple-output (SIMO)-OFDM models has apparently not been studied. In the next chapter, we exploit receive antenna diversity and develop a blind identification technique using the frequency-domain signals.

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<sup>4</sup>Blind channel estimation for OFDM without redundancy has been addressed in [27].

## Chapter 2

# Blind Channel Estimation using Receive Space Diversity

### 2.1 Introduction

The use of multiple receive antennas has been largely in the context of diversity combining, e.g., [33, 34], where the channel is assumed known, or is estimated via pilot symbols, or spatial diversity is used only for beamforming and/or channel tracking. Blind channel estimation of SIMO-OFDM models has apparently not been studied. In this report, we exploit receive antenna diversity and develop a blind identification technique using the frequency-domain signals. Our method possesses the following attractive properties:

- It is performed in the frequency domain. Therefore, unlike the time-domain methods, one can control the number and frequencies of the subcarriers to be used in channel estimation. We will later show how to select, for a fixed number of subcarriers, the optimal set of subcarriers in terms of estimation accuracy. On the other hand, properly designed pre-FFT equalizers could reduce the number of required FFT chipsets.
- Unlike the existing methods, it does not suffer from the self noise effect when the symbol constellations are constant modulus.
- It is SOS-based unlike the method in [32] which uses higher-order statistics. For example, for MPSK symbols, the method in [32] requires  $M^{\text{th}}$ -order statistics. Its performance deteriorates with  $M$  whereas that of the proposed method is independent of  $M$ . Further, our method can cope with a larger delay spread, i.e.,  $L \leq (K - 1)/2$  where  $K$  is the number of modulated subcarriers, compared with the method in [32] for which  $L \leq (K - 1)/M$ . The latter condition might not be satisfied for large constellation sizes.
- It is robust to channel order over-determination.

## 2.2 Signal Model

In this chapter, the  $\ell$ th coefficients for the  $i$ th channel is denoted by  $h_i(\ell)$ . After discarding the cyclic prefix, FFT processing, and ignoring the null-subcarriers (NSC's), the frequency-domain  $n$ th OFDM block at the  $i$ th receive antenna can be, using the signal model presented in Chapter 1, written as

$$\mathbf{y}_i(n) = \mathbf{H}_i \mathbf{s}(n) + \boldsymbol{\eta}_i(n), \quad n = 1, \dots, M, \quad i = 1, \dots, I, \quad (2.1)$$

where  $\mathbf{y}(n)$  is the  $(K \times 1)$  vector whose entries are  $y(n, k)$ ,  $k \in \mathcal{K}$ ,  $\boldsymbol{\eta}(n)$  is additive noise, with  $\mathbf{H}_i = \text{diag}(H_{i,k}, k \in \mathcal{K})$ , and

$$H_{i,k} = \sum_{\ell=0}^L h_i(\ell) \rho_k^{-\ell}, \quad \rho_k := e^{j2\pi k/N}.$$

In the above model, we have assumed the  $\beta_n$ 's defined in Chapter one to be identical across the blocks and they are normalized (with respect to the noise power) to unity

*Notation.* We will let superscript  $*$ ,  $\mathcal{H}$  and  $^\dagger$  denote conjugate, conjugate transpose, and pseudo-inverse;  $\delta_{mn} = 1$  if  $m = n$  and 0 otherwise;  $\delta_{ijk\ell} = 1$  if  $i=j=k=\ell$  and zero otherwise;  $\mathbf{I}_K$  is the  $K \times K$  identity matrix.

We will assume that *i*) the input sequence  $\mathbf{s}(n)$  is zero mean and  $E\{\mathbf{s}(n)\mathbf{s}^H(m)\} = \delta_{nm}\mathbf{I}_K$ , and *ii*) the additive noise is uncorrelated with the data sequence, spatially and temporally white and circularly symmetric Gaussian (STCWG) with variance  $\sigma^2$ .

## 2.3 Identifiability Conditions

The cross-correlation between  $y_i(n, k)$  and  $y_j(n, k)$ , i.e., between the  $k$ -th subcarriers at antennas  $i$  and  $j$  is given by

$$\gamma_{ij}(k) := E\{y_i(n, k)y_j^*(n, k)\} = H_{i,k}H_{j,k}^* + \sigma^2\delta_{ij}, \quad (2.2)$$

with  $k \in \mathcal{K}$ , and  $i, j = 1, \dots, I$ , and where we have used the fact that  $E\{|s(n, k)|^2\} = 1$ .

### 2.3.1 Identifiability of the Additive Noise Power

If the additive noise power is unknown, it can be estimated by exploiting receive diversity. Two diversity branches are sufficient to identify  $\sigma^2$ . Indeed, using  $\gamma_{ij}(k)$ ,  $i \neq j$ , we readily obtain the following equation

$$\sigma^4 - [\gamma_{ii}(k) + \gamma_{jj}(k)]\sigma^2 + \gamma_{ii}(k)\gamma_{jj}(k) - |\gamma_{ij}(k)|^2 = 0.$$

Thus,  $\sigma^2$  can be obtained as the unique solution to a quadratic (only one root is valid). Combining all the correlations, we can obtain

$$\sigma^2 = \frac{1}{K} \sum_{k \in \mathcal{K}} \frac{\alpha(k) - \sqrt{\alpha(k)^2 - 4\beta(k)}}{2}$$

$$\text{where } \alpha(k) = \frac{1}{I(I-1)} \sum_{i,j=1; i \neq j}^I [\gamma_{ii}(k) + \gamma_{jj}(k)]$$

$$\text{and } \beta(k) = \frac{1}{I(I-1)} \sum_{i,j=1; i \neq j}^I [\gamma_{ii}(k)\gamma_{jj}(k) - |\gamma_{ij}(k)|^2].$$

Note that two diversity branches are sufficient, and that  $\sigma^2$  can be identified regardless of the values of the channel coefficients, the  $H_i(\rho_k)$ 's. Here, for simplicity, we focus on the case of temporally and spatially uncorrelated noise keeping in mind that most of the results established here can be extended to the case of temporally correlated but spatially uncorrelated noise.

### 2.3.2 Channel Identifiability

The first term of the RHS of eq. (2.2) can be expressed, using the z-transform, as

$$H_{i,k}H_{j,k}^* = G_{ij}(z)|_{z=\rho_k}$$

where  $G_{ij}(z) = H_i(z)H_j^*(1/z^*)$  and  $H_i(z) = \sum_{\ell=0}^L h_i(\ell)z^{-\ell}$ . The inverse z-transform of  $R_{ij}(z) = G_{ij}(z) + \sigma^2$  is given by

$$r_{ij}(\tau) = \sum_{l=\max(0, -\tau)}^{\min(L, L-\tau)} h_i(l+\tau)h_j^*(l) + \sigma^2\delta_{ij}\delta(\tau), \quad (2.3)$$

with  $\tau = -L, \dots, L$ .

Let  $\mathbf{R}_{ij}$  be the Toeplitz matrix with  $(m, n)$  entry  $r_{ij}(m-n)$ ; then,

$$\mathbf{R}_{ij} = \mathcal{H}_i \mathcal{H}_j^H + \sigma^2 \delta_{ij} \mathbf{I}_{L+1} \quad (2.4)$$

where  $\mathcal{H}_i$  is the  $(L+1 \times 2L+1)$  Toeplitz matrix with first row  $[h_i(0), \dots, h_i(L), 0, \dots, 0]^T$  and first column  $[h_i(0), 0, \dots, 0]^T$ . Concatenating the  $\mathbf{R}_{ij}$ 's, we obtain the following  $I(L+1) \times I(L+1)$  matrix  $\mathbf{R} = \{\mathbf{R}_{ij}\}_{i,j=1}^I$  which can be expressed as

$$\mathbf{R} = \mathcal{H} \mathcal{H}^H + \sigma^2 \mathbf{I}_{I(L+1)} \quad (2.5)$$

where  $\mathcal{H} = [\mathcal{H}_1^T \dots \mathcal{H}_I^T]^T$ . Notice that  $\mathbf{R}$  is the same as the theoretical covariance matrix of a single carrier SIMO system. This implies that all the existing SOS-based channel identification methods could be applied to our matrix  $\mathbf{R}$  provided the latter can be uniquely estimated from the  $\gamma_{ij}(k)$ 's. This condition is satisfied if the number of modulated sub-carriers  $K > 2L$ .

If  $\mathbf{R}$  is uniquely identified, then the issue of channel identifiability is similar to that of single carrier SIMO systems. The conditions for data recovery<sup>1</sup> are however different from

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<sup>1</sup>By data recovery we mean that in the absence of noise, the data streams can be exactly determined up to a sign ambiguity.

those of single carrier SIMO systems (without cyclic prefix).<sup>2</sup> In the literature, channel identifiability is often amalgamated with data recovery. Here, we distinguish between the two concepts. The following Proposition states the channel identifiability conditions when SOS are used.

**Proposition 1.** *The  $(L+1)$ -tap channels of an  $I$  receive-diversity OFDM system, in the presence of STCWG noise, can be blindly identified using second-order statistics (up to a phase ambiguity) if the number of modulated subcarriers  $K > 2L$ , and any roots that are common to all the channels are on the unit circle.*

**Proof.** Once the  $R_{ij}(z)$ 's and  $\sigma^2$  are identified, the  $G_{ij}(z)$ 's can be determined as  $G_{ij}(z) = R_{ij}(z) - \sigma^2$ . If  $\theta$  is a root of  $G_{ii}(z)$  then its reciprocal  $1/\theta^*$  is also a root of  $G_{ii}(z)$ . This implies that identifying  $G_{ii}(z)$  leads to  $2^{q(i)-q_1(i)}$  spectrally equivalent channels  $H_i(z)$ , where  $q(i)$  is the total number of roots  $H_i(z)$ <sup>3</sup> and  $q_1(i)$  is the number of unit-modulus roots of  $H_i(z)$ . The ambiguity in the non unit-modulus (NUM) roots can be eliminated using receive diversity as shown next. Note that at least one of the  $h_i(0)$ 's has to be non-zero to account for the inherent delay ambiguity in blind techniques.

For simplicity, we assume that  $h_i(0), h_j(0) \neq 0$ <sup>4</sup>. If  $\xi_{i,l}$  and  $\xi_{j,l}$  denote the roots of  $H_i(z)$  and  $H_j(z)$  respectively, then

$$G_{ij}(z) = h_i(0)h_j^*(0) \prod_{l=1}^{q(i)} [1 - \xi_{i,l}z^{-1}] \prod_{l=1}^{q(j)} [1 - \xi_{j,l}^*z]$$

Therefore identifying the NUM roots of  $G_{ij}(z)$  will provide the set of roots  $\{\xi_{i,l}, 1/\xi_{j,m}^*\}$ ,  $l = 1, \dots, q_1(i)$ ;  $m = 1, \dots, q_1(j)$ . If  $H_i(z)$  and  $H_j(z)$  share no common NUM roots, then the channels  $H_i(z)$  and  $H_j(z)$  can be blindly identified (up to a phase ambiguity) using  $G_{ij}(z)$ ,  $G_{ij}(z)$  and  $G_{jj}(z)$ . More generally, if no NUM root is common to *all* the channels, then all the channels can be uniquely identified (up to a phase ambiguity) using  $H_{ij}$ ,  $i, j = 1, \dots, I$ . These results are also valid for single carrier SIMO systems provided the covariance matrix is uniquely identified.  $\square$

Note that for single-carrier systems, channel identifiability is guaranteed under the same conditions as above except for the condition  $K > 2L$  which is not applicable.

### 2.3.3 Data Recovery

For single carrier systems, data recovery is guaranteed if the matrix  $\mathcal{H}$  is full column rank, which is obtained if the channels share no common zeros. For our OFDM system, data recovery is guaranteed under a weaker condition as stated next.

**Proposition 2.** *For an  $I$  receive-diversity OFDM system operating over a  $(L+1)$ -tap channel, in STCWG noise, data recovery is guaranteed using second-order statistics (up*

<sup>2</sup>Data recovery conditions and channel estimation for cyclic-prefixed single carrier systems are similar to those of our OFDM system.

<sup>3</sup>if  $h_i(1), h_i(L) \neq 0$  then  $q(i) = L$

<sup>4</sup>We can check whether  $h_j(0) \neq 0$  by inspecting the  $r_{ij}$ 's in eq.(2.3).

to a phase ambiguity) if the number of modulated subcarriers  $K > 2L$ , any roots that are common to all the channels are on the unit circle, and the common unit-modulus roots, if any, do not coincide with the modulated subcarrier frequencies,  $2\pi k/N$ ,  $k \in \mathcal{K}$ .

**Proof.** The first conditions are required for channel identifiability. Once the channels have been identified, their frequency responses at the subcarriers frequencies are calculated. If the channels have a common root that is constant modulus and its phase coincides with one of the subcarriers frequencies, then the corresponding frequency responses will be zero at this subcarrier, in which case data cannot be recovered.<sup>5</sup>  $\square$

## 2.4 Channel Estimation Approach

Channel estimation is based on the estimation of the matrix  $\mathbf{R}$ , which can be done either in the time-domain, or based on the frequency domain correlations, the  $\gamma_{ij}$ 's. Here, we use the latter approach.

We first estimate the  $\gamma_{ij}(k)$ 's for  $k \in \mathcal{K}$  and  $i, j = 1, \dots, I$ ,

$$\hat{\gamma}_{ij} = \frac{1}{M} \sum_{n=1}^M \mathbf{y}_i(n) \odot \mathbf{y}_j^*(n), \quad i, j = 1, \dots, I \quad (2.6)$$

where  $\odot$  denotes the Shur-Hadamard product (i.e., element wise multiplication). Estimates of the  $\mathbf{r}_{ij} = [r_{ij}(-L), \dots, r_{ij}(L)]^T$  are obtained via

$$\hat{\mathbf{r}}_{ij} = \mathbf{B}_{\mathcal{K}}^\dagger \hat{\gamma}_{ij} \quad i, j = 1, \dots, I \quad (2.7)$$

where  $\mathbf{B}_{\mathcal{K}}$  is the  $K \times (2L + 1)$  matrix whose elements are  $\exp(j2\pi kl/N)$  with  $k \in \mathcal{K}$ , the set of active subcarriers, and  $l = -L, \dots, L$ .

For constant-modulus symbols and in the absence of noise,  $\hat{\gamma}_{ij}(k) = \gamma_{ij}(k)$  so that  $\hat{\mathbf{R}}_{ij} = \mathbf{R}_{ij}$ . This implies that for PSK constellations, the channels can be exactly identified (up to a phase ambiguity) using a single OFDM block in the absence of noise. Channel identification using the time domain-based estimation of covariance matrix  $\mathbf{R}$  does not benefit from this nice property. It is this property which makes the proposed method particularly efficient for PSK-OFDM. (In the absence of noise, it was shown in [35] that both channel and symbols can be recovered from a single OFDM symbol with  $m$ -PSK modulation if # sub-carriers  $N > mL$ .) For non-constant modulus symbols, e.g., QAM constellations,  $M$  is required to be moderate/large in order for  $\hat{\gamma}_{ij}(k)$  to be a good estimate of  $\gamma_{ij}(k)$  even in the absence of noise. This is referred to as the self noise effect.

## 2.5 Performance Analysis

Here, we derive the covariances of the cross-correlation estimates. Once these covariances are obtained, performance analysis of channel estimates can be carried out along the lines

<sup>5</sup>Note however that coding across the subcarriers is always used in order to address this problem.



of [36]. Because the estimation of the cross-correlations is carried out in the frequency domain, we are able to provide closed-form expressions for the *finite-sample* covariances of the cross-correlations.

### 5.1 Channel-dependent bounds

The finite-sample covariances of the sample cross-correlation  $\gamma_{i,j}$ , based on  $M$  OFDM blocks, are given by

$$M \text{cov}(\hat{\gamma}_{i,j}, \hat{\gamma}_{k,\ell}) = (\mu_{4s} - 1) \mathbf{H}_i \mathbf{H}_j^H \mathbf{H}_k^H \mathbf{H}_\ell + \sigma^2 \delta_{j\ell} \mathbf{H}_i \mathbf{H}_k^H \\ + \sigma^2 \delta_{ik} \mathbf{H}_j^H \mathbf{H}_\ell + (\delta_{ijk\ell} c_{4\eta} + \delta_{ik} \delta_{j\ell}) \sigma^4 \mathbf{I}_K$$

where  $\mu_{4s} = E\{|s(n, m)|^4\} / (E\{|s(n, m)|^2\})^2$ , and  $c_{4\eta} := -2 + E\{|\eta|^4\} / \sigma^4$  is the (normalized) fourth-order cumulant. Note that  $\text{cov}(\hat{\gamma}_{i,j}, \hat{\gamma}_{k,\ell}^*) = \text{cov}(\hat{\gamma}_{i,j}, \hat{\gamma}_{\ell,k})$  since  $\gamma_{k,\ell}(\cdot) = \gamma_{\ell,k}(\cdot)^*$ .

For constant-amplitude symbols, e.g., PSK constellations,  $\mu_{4s} = 1$ , and the first term on the RHS of the above equation vanishes. This implies that for PSK symbols, the cross-correlations estimates are asymptotically (i.e., as  $\sigma^2 \rightarrow 0$ ) consistent. This is in contrast with the case of QAM constellations for which  $\mu_{4s} \neq 1$ . The corresponding cross-correlation estimates are consistent only when the number of blocks  $M$  tends to  $\infty$ . Note also that  $(\mu_{4s} - 1)$  increases with the constellation size, and so do the errors on the cross-correlation estimates. For instance,  $\mu_{4s} = 1.32$  for 16QAM and  $\mu_{4s} = 1.38$  for 64QAM (with a limiting value of 1.4). Using eq. (2.7), the covariances of the sample estimates of  $\mathbf{r}_{ij}$  are obtained as

$$\text{cov}(\hat{\mathbf{r}}_{i,j}, \hat{\mathbf{r}}_{\ell,k}^*) = \text{cov}(\hat{\mathbf{r}}_{ij}, \hat{\mathbf{r}}_{k,\ell}) = \mathbf{B}_K^\dagger \text{cov}(\hat{\gamma}_{i,j}, \hat{\gamma}_{k,\ell}) (\mathbf{B}_K^\dagger)^H. \quad (2.8)$$

### 5.2 Channel-independent bounds

These covariance expressions are channel dependent. As wireless channels are random, it is interesting to average the covariance expressions over all the possible channels. We assume that the channels are independent and Rayleigh distributed (the Ricean case could be treated in the same manner). Let  $\mathbf{Q}$  denote the covariance matrix of the channel coefficients, the  $\mathbf{h}_i$ 's. For simplicity, we focus on the case of uncorrelated scattering, for which  $\mathbf{Q}$  is diagonal; then, with  $\sigma_H^2 = \text{Tr}\{\mathbf{Q}\}$ , we obtain

$$E\{H_{i,m} H_{j,m}^* H_{k,m}^* H_{\ell,m}\} = \sigma_H^4 [\delta_{ij} \delta_{k\ell} + \delta_{ik} \delta_{j\ell}].$$

In the case of uncorrelated scattering, the averaged finite-sample covariances of the sample cross-correlation  $\gamma_{i,j}$  are

$$\text{cov}(\hat{\gamma}_{i,j}, \hat{\gamma}_{k,\ell}) = \alpha_{ijk\ell} \mathbf{I}_K, \quad \text{where}$$

$$\alpha_{ijk\ell} = \sigma_H^4 (\mu_{4s} - 1) [\delta_{ij} \delta_{k\ell} + \delta_{ik} \delta_{j\ell}] + 2\sigma_H^2 \sigma^2 \delta_{ik} \delta_{j\ell} \\ + \sigma^4 \delta_{ik} \delta_{j\ell} + \delta_{ijk\ell} c_{4\eta}.$$

The averaged errors on the estimation of the  $\mathbf{r}_{ij}$  are given by

$$\text{cov}(\hat{\mathbf{r}}_{ij}, \hat{\mathbf{r}}_{kl}) = \alpha_{ijkl} \mathbf{B}_{\mathcal{K}}^{\dagger} (\mathbf{B}_{\mathcal{K}}^{\dagger})^{\mathbf{H}}.$$

### 5.3 Optimal Selection of Subcarriers

In order to reduce the computational complexity of the proposed channel identification, it is desirable to reduce the number of subcarriers used for the estimation of the cross-correlation coefficients. In this subsection, we answer the following question: for a given number of subcarriers to be used in blindly estimating the channels, what is the optimal set of subcarriers in terms of estimation accuracy.

In the previous subsection, we have shown that under the uncorrelated scattering scenario, the errors in the estimation of the  $\gamma_{ij}$  are independent of the frequencies of the subcarriers set. The errors in the estimation of the  $\mathbf{r}_{ij}$  do however depend on  $\mathcal{K}$ .

**Proposition 3:** *The optimal selection of  $D$  subcarriers in terms of channel estimation performance is given by*

$$\tilde{\mathcal{D}} = \arg \min_{\mathcal{D} \subset \mathcal{K}} \text{Tr} \left\{ \mathbf{B}_{\mathcal{D}}^{\dagger} (\mathbf{B}_{\mathcal{D}}^{\dagger})^{\mathbf{H}} \right\}. \quad (2.9)$$

Thus, if  $D \leq K/2$ , then the optimal set  $\tilde{\mathcal{D}}$  is obtained by selecting equally spaced subcarriers with maximum spacing between the subcarriers.  $\square$

## 2.6 Channel Estimation Algorithms

Given estimates of  $\hat{\mathbf{r}}_{ij}$ , the noise power can be estimated using results in Sec 3.1.

**Cross-Correlation Matching Method:** Let  $\bar{\mathbf{r}}_{ij} = \mathbf{r}_{ij} - \sigma^2 \delta_{ij} \mathbf{e}_{L+1}$  and  $\hat{\bar{\mathbf{r}}}_{ij} = \hat{\mathbf{r}}_{ij} - \hat{\sigma}^2 \delta_{ij} \mathbf{e}_{L+1}$ , where  $\mathbf{e}_{L+1}$  is a  $(2L+1)$ -element vector whose  $(L+1)$ th element is one, and the rest are zeros. Now, we propose to estimate the channels using the following matching criterion

$$J(\mathbf{h}) = \sum_{i,j=1; i \leq j}^I |\hat{\bar{\mathbf{r}}}_{ij} - \bar{\mathbf{r}}_{ij}|_{\mathbf{W}_{ij}} \quad (2.10)$$

where  $\mathbf{W}_{ij}$  are weighting matrices and  $|\mathbf{e}|_{\mathbf{W}_{ij}} = \mathbf{e}^H \mathbf{W}_{ij} \mathbf{e}$  for any vector  $\mathbf{e}$  of appropriate dimensions. The optimal weighting matrices can be obtained using eq. (2.8). The performance of CCM can be derived along the lines of [36], which will then serve as a benchmark for other SOS-based methods.

**Subspace Method:** Using eq. (2.5), we can readily adapt the subspace approach of [37] to obtain channel estimates. Note the slightly relaxed conditions of Propositions 1 and 2 in the context of multi-carrier signals. Note that the deterministic subspace method does not suffer from self noise even for QAM constellations. However, the identifiability condition for these methods is that the channels have no common root, which is more restrictive than the identifiability condition established above. Moreover, Deterministic subspace methods fail if the channel order is not estimated properly.

**Root Selection Method:** The proof of Proposition 1 outlines the method. From  $G_{ij}$  we obtain the roots of  $H_1(z)H_2(1/z^*)$ . Every root of  $G_{12}$  can be compared with those of  $G_{11}$ , and the closest selected. The set of  $L$  roots of  $G_{11}$  closest to those of  $G_{12}$  yields  $H_1(z)$ .  $H_2(z)$  is obtained similarly.

## 2.7 Simulations

Since the subspace for SIMO models is well established, we focus on the novel method: the root selection method.

To test the performance of the proposed algorithm, a suite of simulations were run, with the following set of parameters, unless otherwise stated: total # sub-carriers,  $N = 64$ ; # virtual sub-carriers (NSC's) 15; SNR = 15 dB; 3-ray channel with independent Rayleigh fades, exponential power delay profile with decay parameter  $\beta = 1/5$ ; 2 receive antennas;  $M = 10$  OFDM blocks for channel estimation. Bias and MSE were estimated from a suite of 500 Monte Carlo runs; channel parameters were drawn independently for each run. Mean bias and MSE, for the set of  $L + 1$  channel taps are reported; in order to fix the scalar gain ambiguity - present with any blind channel estimation method - we normalized the channel response to unit norm. The symbol sets tested were 16-QAM, 64-QAM, QPSK, 8-PSK, 16-PSK, and 64-PSK. Solid lines correspond to the PSK symbol sets, and dashed lines to the QAM symbol sets.

Figure 2.1 shows performance vs. SNR; notice that QAM performance has the usual floor effect; PSK performance does not suffer from this, and PSK performance does not depend upon the size of the alphabet. Figure 2.2 shows performance as the number of OFDM symbols was increased; with PSK symbol sets good performance is achieved even with 5-10 OFDM symbols. Figure 2.3 shows results as the actual channel length was increased (with  $N = 64$ , and 15 NSC's; adequate CP was used in every case). Note that degradation in performance is graceful. Next, keeping  $L = 3$  and  $N = 64$  fixed, the number of band-edge NSC's was increased; figure 2.4 indicates that degradation is graceful.

## 2.8 Discussion

From the simulations, we verify that performance for PSK symbol sets does not suffer from the floor effect, and that good channel estimates are obtained with as few as 5 to 10 OFDM symbols.

In the absence of CP, spatial diversity has been exploited for estimating channel parameters; comparing with results in [38], we note that the absence of CP cannot be easily made up with spatial diversity. Spatial diversity (here, the inclusion of one extra receive antenna) tremendously decreases the number of OFDM symbols required for reliable channel estimation by SISO approaches (e.g., the cyclic approach of [28] and the

finite-alphabet approach of [32] both require substantially more OFDM symbols).

The performance of the estimators proposed in Section 6 can be readily accomplished using the results of Section 5, along the lines of [36].

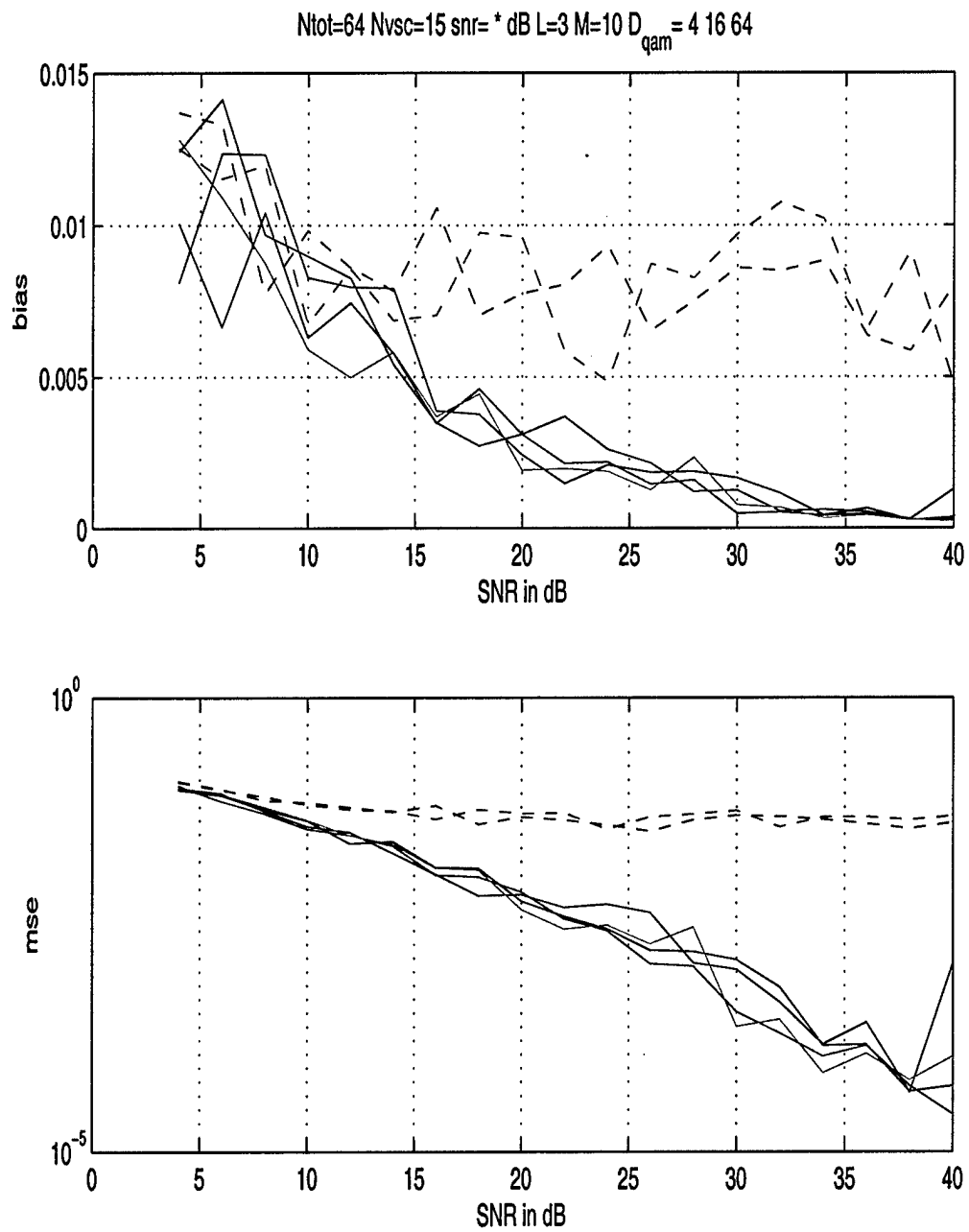


Figure 2.1: Bias and MSE vs. SNR

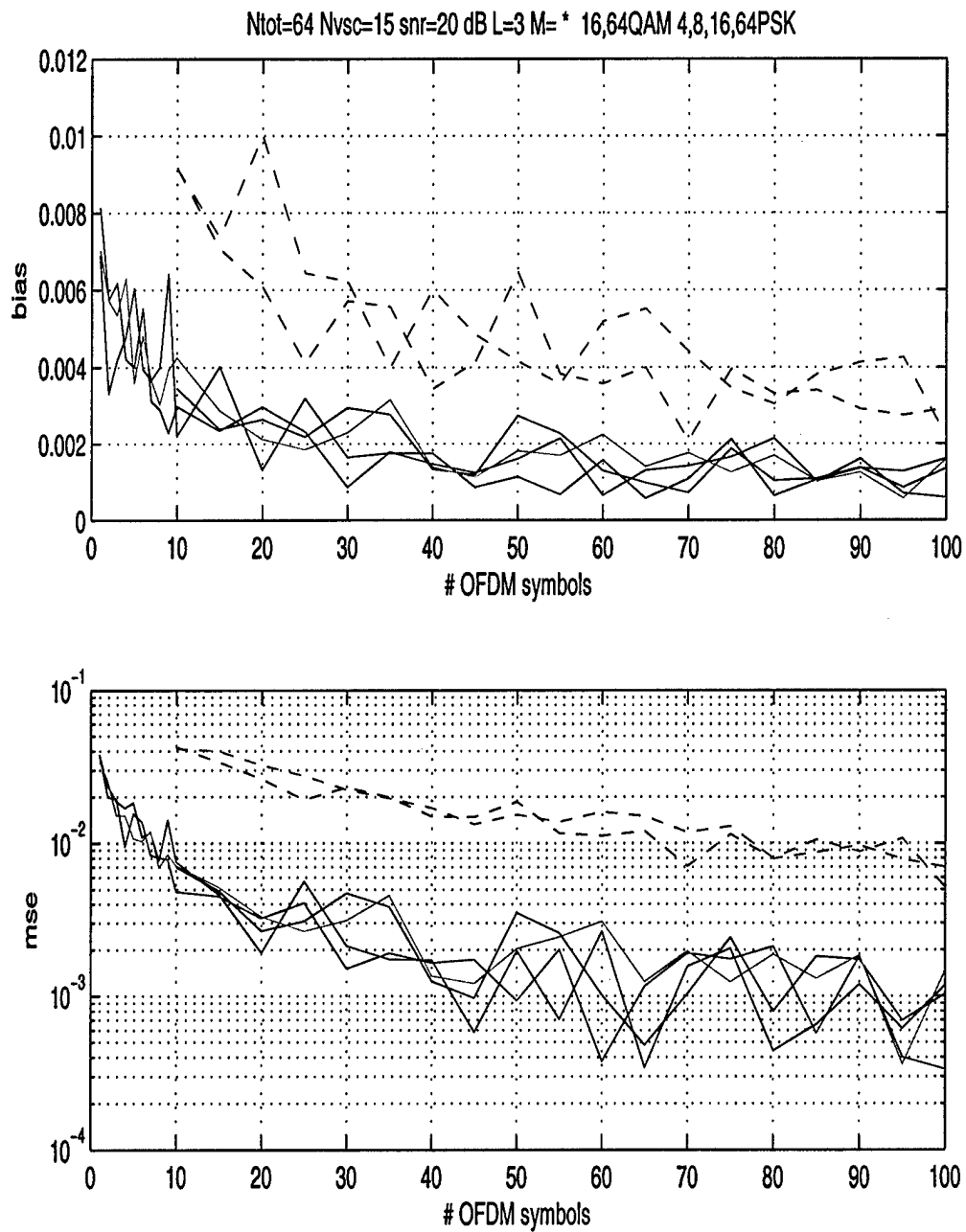


Figure 2.2: Bias and MSE vs. # OFDM symbols

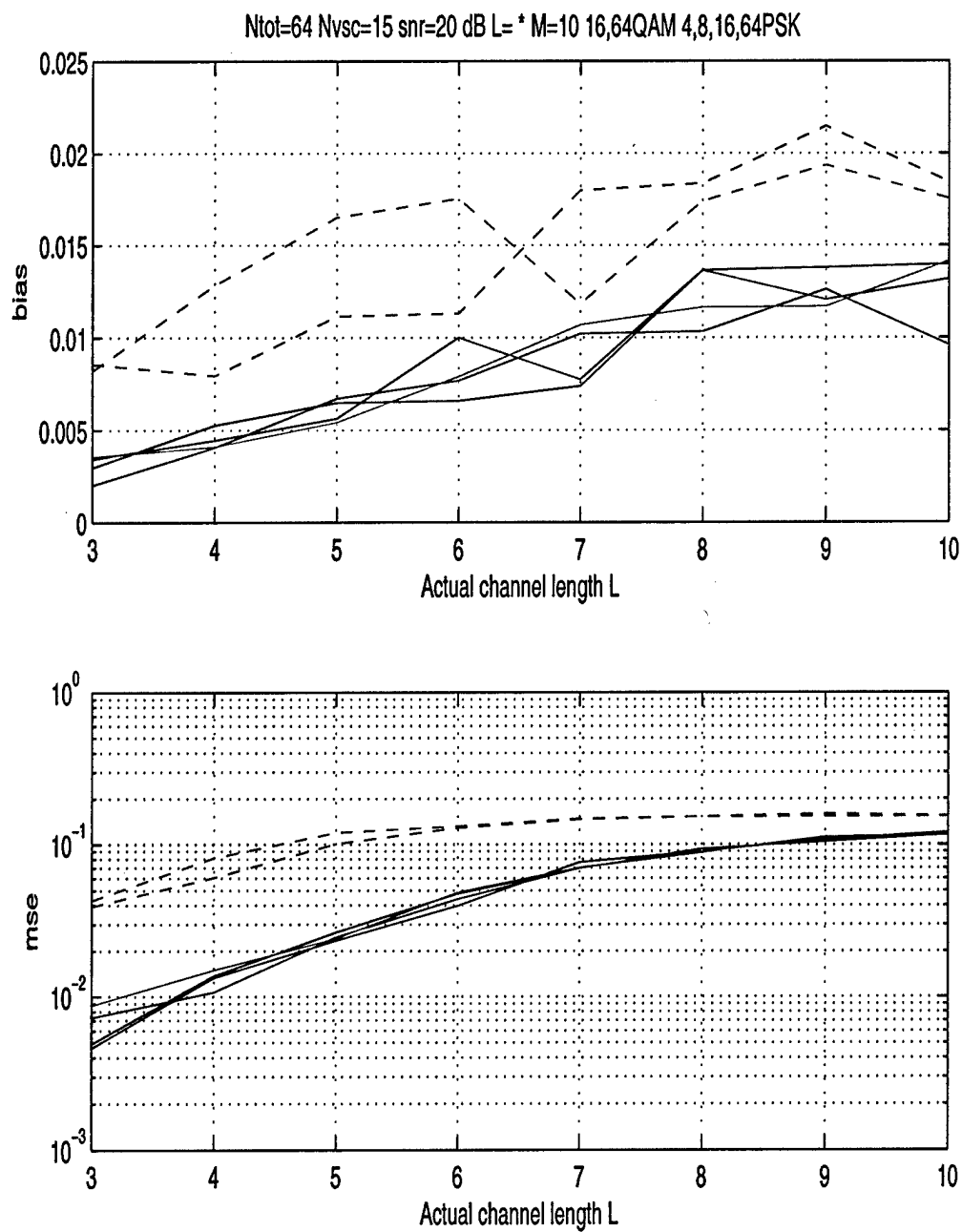


Figure 2.3: Bias and MSE vs. Channel length

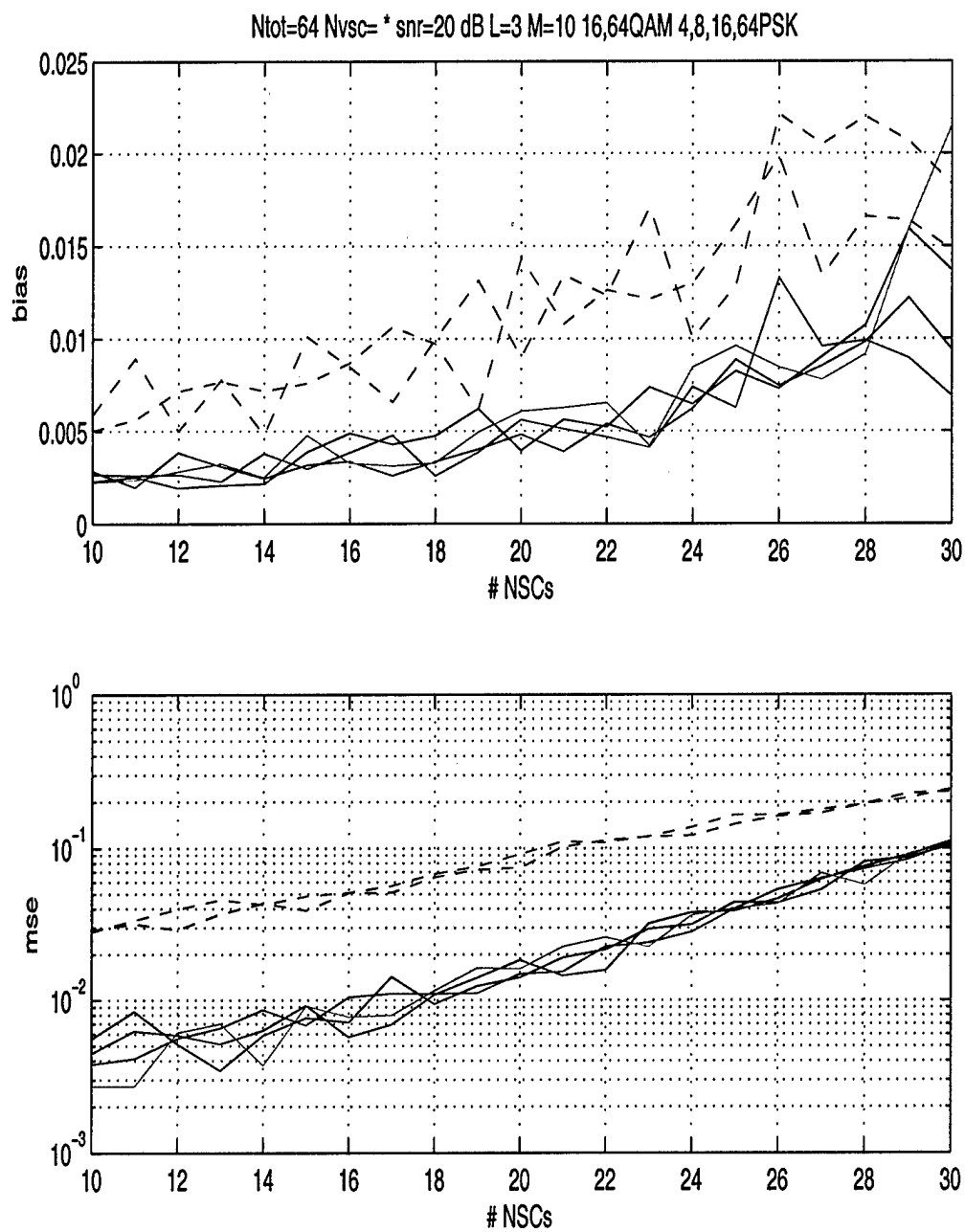


Figure 2.4: Bias and MSE vs. # null sub-carriers



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